Class 11

ALL TOPICS NUMERICALS

Thermodynamics

FOR PRE-BOARDS & FINAL

Numerical Problems based on

$$Q = m\,C_v\,\Delta T$$

Numerical Problem #1

The specific heat of gold is 129 J/kg·K. What is the quantity of heat energy required to raise the temperature of 100 g of gold by 50 K?

$$Q = m C_v \Delta T$$

$$Q = (0.1\ kg \times 129\ J\ kg^{-1} K^{-1} \times 50\ K)$$

$$Q = 129 \times 5\ J$$

$$\boxed{Q = 645\ J}$$

Numerical Problem #2

A geyser heats water flowing at the rate of 3.0 litres per minute from 27 °C to 77 °C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0×10^4 J/g? Given specific heat capacity of water = 4.2 J g^{-1} C^{-1}

$$\frac{Q}{t} = \frac{m}{t} c_v \Delta T = \frac{3000\,g}{1\,min} \times \frac{4.2\,J}{1\,g\,C} \times 50°C$$

$$\frac{Q}{t} = 3000 \times 42 \times 5 \frac{J}{min} = 63 \times 10^4 \frac{J}{min}$$

$$\text{Rate of consumtion (in g/min)} = \frac{63 \times 10^4\ J/min}{4.0 \times 10^4\ J/g} = 15.75\ g/min$$

Numerical Problems based on Calorimetry

Numerical Problem #3

10g water at 80° C is mixed with 50 g copper at 10° C. Find the final temperature of mixture.
heat capacity of water 4.2J /gC and of copper is 0.4J /gC

water	copper
$m = 10g$	$m = 50g$
$c_v = 4.2\ J/gC$	$c_v = 0.4\ J/gC$
$\Delta T = 80 - \theta$	$\Delta T = \theta - 10$
$\theta = ?$	

Heat lost by water = Heat gained by copper

$$10 \times 4.2 \times (80 - \theta) = 50 \times 0.4 \times (\theta - 10)$$

$$42(80 - \theta) = 20(\theta - 10)$$

$$3360 - 42\theta = 20\theta - 200$$

solving the above eqn, you get $\theta = 57.41°C$

Numerical Problem #4

A copper block of mass 5.0 kg is heated to a temperature of 500° C and is placed on a large ice block. What is the maximum amount of ice that can melt?

Specific heat of copper : $0.39\ J\ g^{-1}C^{-1}$

Latent heat of fusion of water : $335\ J\ g^{-1}$

max Heat Lost by copper = max Heat gained by ice

$$m\,C_v\,\Delta T = M\,L$$

$$5\times 1000 \times 0.39 \times 500 = M \times 335$$

$$5\times 1000 \times 39 \ \cancel{\times 5} = M \times \cancel{335}\ 7$$

$$M = \frac{195}{7} \times 1000\ g = \frac{195}{7}\ kg = 27.85\ kg$$

Numerical Problem #5

Do it yourself

The temperature of 170g of water at 50° C is lowered to 5° C by adding a certain amount of ice to it. Find the mass of ice added.

Given:

Specific heat capacity of water = 4200 J kg^{-1} C^{-1} and Specific latent heat of ice = 336000 J/ kg

water ice

$m c_v \Delta T = M L_f$

Numerical Problem #6

Do it yourself

A solid of mass 50 g at 150° C is placed in 100 g of water at 11° C, when the final temperature recorded is 20°C. Find the specific heat capacity of the solid.
(Specific heat capacity of water = 4.2 J $g^{-1}C^{-1}$)

water   Solid

$$m c_v \Delta T = M c_v' \Delta T'$$

$$100 \times 4.2 \times (20-11) = 50 \times c_v' \times (150-20)$$

Ans = $0.58\ J g^{-1} C^{-1}$

Numerical Problems based on

1st law of thermodynamics

$$Q = \Delta U + W$$

Q → Heat given to system

ΔU → change in internal energy

W → work done by system

$$dQ = dU + PdV$$

1 Cal = 4.2 J

Numerical Problem #7

Do it yourself

100 g of water is heated from 30° C to 50° C . Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4148 J / kg^{-1} K^{-1})

a)8.4 kJ b)84 kJ c)2.1 kJ d)4.2 kJ

$dQ = dU + PdV$

if $dV = 0$, $dU = dQ = dm C_V \Delta T$

Numerical Problem #8

50 J of heat is given to a system and its internal energy increases by 20 joule. Calculate the work done by the system.

$Q = +50\,J \quad , \quad \Delta U = +20\,J$

$Q = \Delta U + W_{by\ sys}$

$50 = 20 + W_{by\ sys}$

$\boxed{W = 30\,J}$

Numerical Problem #9

80 J of heat is given to a system and its internal energy decreases by 25 joule. Calculate the work done by the system.

$Q = +80\,J$, $\Delta U = -25\,J$

$Q = \Delta U + W_{by\ sys}$

$80 = -25 + W_{by\ sys}$

$W = 105\,J$

Numerical Problem #10

40 J of heat is taken from system and its internal energy increases by 20 joule. Calculate the work done by the system.

$Q = -40\,J$, $\Delta U = +20\,J$

$Q = \Delta U + W_{by\ sys}$

$-40 = 20 + W_{by\ sys}$

$\boxed{W = -60\,J}$

Numerical Problem #11

30 J of heat is taken from system and its internal energy decreases by 20 joule. Calculate the work done by the system.

$Q = -30J$, $\Delta U = -20J$

$Q = \Delta U + W_{by\ sys}$

$-30 = -20 + W_{by\ sys}$

$W = -10J$

Numerical Problem #12

Find the change in internal energy of a gas when it absorbs 80 calories of heat and performs work equal to 170 joule.

$Q = 80\ \text{Cal}$, $W_{\text{by sys}} = 170\ J$

$Q = \Delta U + W \Rightarrow \Delta U = Q - W$

$\Delta U = (80 \times 4.184\ J) - (170\ J)$

$\Delta U = (334.72 - 170)\ J$

$\boxed{\Delta U = 164.72\ J}$

$\boxed{4.184\ J = 1\ \text{cal}}$

Must remember this conversion!

Numerical Problem #13

1 g of water at 373 K is converted into steam at the same temperature. The volume of 1 cm^3 of water becomes 1671 cm^3 on boiling. Calculate change in the internal energy of the system, if heat of vaporisation is 540 cal/g.

Given standard atmospheric pressure = $1.013 \times 10^5\ N/m^2$

$Q = \Delta U + W_{by\ sys.}$

$\Delta U = Q - W = mL - PdV$

$\Delta U = \left(1g \times 540 \frac{cal}{g}\right) - \left(1.013 \times 10^5 \frac{N}{m^2} \times (1671 - 1) \times 10^{-6} m^3\right)$

$\Delta U = (540\ Cal) - (1.013 \times 1670 \times 10^{-1}\ Nm)$

1 Nm = 1 Joule

$\Delta U = (540\ cal) - (1.013 \times 167\ J)$

$\Delta U = 540 \text{ cal} - (1.013 \times 167 \text{ J})$

$\Delta U = 540 \text{ cal} - 169.171 \text{ J}$

$\Delta U = 2259.36 \text{ J} - 169.171 \text{ J}$

$\boxed{\Delta U = 2090.189 \text{ J}}$ Ans

1 cal = 4.184 J
very important conversion

or we can convert Joule into cal, to get answer $\boxed{\Delta U = 499.56 \text{ cal}}$ Ans

Numerical Problem #14

Find the change in internal energy of a gas when it absorbs 20 calories of heat and work of 55 joule is performed on it.

$Q = +20\,Cal$, $W_{on\ system} = 55J$

$W_{by\ system} = -W_{on\ system}$
$= -55J$

$Q = \Delta U + W \Rightarrow \Delta U = Q - W_{by\ system}$

$\Delta U = (20\,Cal) - (-55J) = 20\,Cal + 55J$

$1\,Cal = 4.184\,J$

$\Delta U = 83.68\,J + 55J = 138.68\,J$

Numerical Problems based on

Isothermal and adiabatic processes

Isothermal

$$PV = \text{Constant}$$

$$P_1 V_1 = P_2 V_2$$

Adiabatic

$$PV^{\gamma} = \text{constant}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$TV^{\gamma-1} = \text{constant}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\frac{T^{\gamma}}{P^{\gamma-1}} = \text{constant}$$

$$\frac{T_1^{\gamma}}{P_1^{\gamma-1}} = \frac{T_2^{\gamma}}{P_2^{\gamma-1}}$$

Numerical Problem #15

Find the pressure required to compress a gas adiabatically at atmospheric pressure to one fifth of its volume . GIven $\gamma = 1.4$

$V_f = \frac{1}{5} \times V_i$, $\gamma = 1.4$, $P_i = 1$ atm , $P_f = ?$

$PV^{\gamma} = \text{constant}$ $\Rightarrow P_i V_i^{\gamma} = P_f V_f^{\gamma}$ $\Rightarrow \frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma}$

$\Rightarrow \boxed{P_f = 5^{1.4}}$ Ans

If we want to solve it further, Take log both sides

$\log P_f = 1.4 \times \log 5 = 1.4 \times 0.699 = 0.978$

$\Rightarrow \boxed{P_f = \log^{-1}(0.978) = 9.519 \text{ atm}}$

Numerical Problem #16

Do it yourself

A balloon is filled with hydrogen at room temperature, it will burst if pressure exceeds 0.2 bar. If at 1 bar pressure the gas occupies 2.27 L volume, upto what volume can the balloon be expanded?

Numerical Problem #17

Do it yourself

A gas occupies a volume of 250 ml at 745 mm Hg and 25° C. What additional pressure is required to reduce the gas volume to 200 ml at the same time? 180.25 mm 200.9 mm 186.25 mm 189.4 mm

Numerical Problems based on

Work done in Isothermal and adiabatic processes

Isothermal

$$w = RT \ln\left(\frac{V_2}{V_1}\right)$$

or

$$w = 2.303\, RT \log\left(\frac{V_2}{V_1}\right)$$

Adiabatic

$$w = \frac{R}{1-\gamma}\left[T_2 - T_1\right]$$

Numerical Problem #18

A gas ($\gamma = 1.4$) of 2 m^3 Volume and at a pressure of 4×10^5 N/m^2 is compressed adiabatically to a volume 0.5 m^3. Find its new pressure. Calculate the work done in the process?

Given $4^{1.4} = 6.96$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$4 \times 10^5 \times 2^{1.4} = P_2 \times 0.5^{1.4}$$

$$P_2 = \frac{4 \times 10^5 \times 2^{1.4}}{\left(\frac{1}{2}\right)^{1.4}} = 4 \times 10^5 \times \left(2^{1.4}\right)^2 = 4 \times 10^5 \times 4^{1.4} = 4 \times 10^5 \times 6.96$$

$$P_2 = 2.78 \times 10^6 \frac{N}{m^2}$$

$$W = \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right]$$

$$W = \frac{1}{-0.4} \times \left[2.78 \times 10^6 \times \frac{1}{2} - 4 \times 10^5 \times 2\right]$$

$$W = \frac{-10}{4} \times \left[1.39 \times 10^6 - 8 \times 10^5\right] = -2.5 \times \left[(1.39 \times 10) - 8\right] \times 10^5$$

$$W = -2.5 \times \left[13.9 - 8\right] \times 10^5 = -2.5 \times 5.9 \times 10^5$$

$$\boxed{W = -14.75 \times 10^5 \ J}$$

Numerical Problem #19

Two moles of oxygen at 0°C (273 K) are compressed until the volume remains one-fourth of the initial volume at the same temperature. Calculate the work done.

$R = 8.31\ J\ mol^{-1}\ K^{-1}$. , $\log 2 = 0.3010$

$$V_2 = \frac{V_1}{4} \Rightarrow V_1 = 4V_2$$

$$w = 2.303\, nRT \log\left(\frac{V_2}{V_1}\right) = 2.303 \times 2 \times 8.31 \times 273 \times \log\left(\frac{V_2}{4V_2}\right)$$

$$w = 10449.3 \times \log\left(\frac{1}{4}\right) = 10449.3 \times (\log 1 - \log 4) = 10449.3 \times (0 - \log 2^2)$$

$$w = 10449.3 \times -2\log 2 = 10449.3 \times -2 \times 0.3010$$

$$w = -6290.48\ J$$ Ans

Numerical Problem #20

Do it yourself

An ideal gas of volume 1 litre and at pressure 8 atmospheres expands adiabatically until the pressure drops to 1 atmosphere. Find the final volume and work done.

Given : $\gamma = 1.5$, 1 atmosphere $= 1.013 \times 10^5\ Nm^{-2}$ and 1 litre $= 10^{-3}\ m^3$.

for Adiabatic Process,

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\& \quad W = \frac{1}{1-\gamma}\left[P_2 V_2 - P_1 V_1\right]$$

Answers! -

$V_2 = 4$ Litre

$W = 810.4\ J$

Numerical Problem #21

Do it yourself

One mole of oxygen at NTP is compressed adiabatically to 5 atmospheres. What is the new temperature and the work done?

Given : $\gamma = 1.4$, $R = 8.31\ J\ mol^{-1}\ K^{-1}$.

for Adiabatic Process ,

$$\frac{T_1^{\gamma}}{P_1^{\gamma-1}} = \frac{T_2^{\gamma}}{P_2^{\gamma-1}}$$

$$W = \frac{R}{1-\gamma}\left[T_2 - T_1\right]$$

Answers! -

$T_2 = 432\ K$

$W = -3299\ J$

Numerical Problem #22

Do it yourself

0.5 mole of gas at temperature 300 K expands isothermally from an initial volume of 2.0 L to final volume of 6.0L.

(a) What is the work done by the gas ? (R = 8.31 J mol^{-1} K^{-1}.)

(b) How much heat is supplied to the gas ?

(c) What is final pressure of gas ?

Numerical Problems based on

Heat Engine

Skip if not included

$$\eta = 1 - \frac{T_2}{T_1}$$

T_2 → sink Temp. (in kelvin)

T_1 → source Temp. (in kelvin)

where $T_1 > T_2$

or $\eta = +ve$

or

$$\eta = 1 - \frac{Q_2}{Q_1} \quad \text{or} \quad \eta = \frac{W}{Q_1}$$

Numerical Problem #23

Calculate the efficiency of a reversible heat engine workig b/w the temperature 100° C and 0° C

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{373} = \frac{100}{373}$$

$$\eta\% = \frac{100}{373} \times 100 = \frac{10000}{373} = 26.8\%$$

Numerical Problem #24

Do it yourself

Two engines A and B are working at the following source and sink temperatures.

	Source temperature	Sink temperature
Engine A	400K	350K
Engine B	350K	300K

Which engine is more efficient , and by how much?

Numerical Problem #25

A Carnot engine whose sink is at 290 K has an efficiency of 30%. By how much the temperature of the Source be increased to have its efficiency equal to 50%, keeping sink temperature constant.

$$\frac{30}{100} = 1 - \frac{290}{T_1}$$

$$0.3 = 1 - \frac{290}{T_1}$$

$$\frac{290}{T_1} = 0.7$$

$$T_1 = 414.29\,K$$

$$\frac{50}{100} = 1 - \frac{290}{T_1'}$$

Solving this, we get $T_1' = 580\,K$

$$\Delta T = 580K - 414.29\,K$$

$$\Delta T = 165.71\,K$$